CASIO

Higher Order Polynomial Functions

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LEVEL
High schools when students doing polynomial functions.

OBJECTIVES
To explore polynomial functions of degree 2, 3 and 4 with a graphics calculator.

CORRESPONDING eActivity
HIGHPOLY.g1e

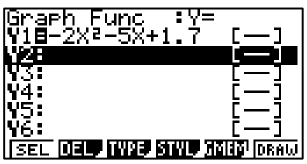
OVERVIEW
This activity explores some higher order polynomial functions which are usually quite challenging without the aid of technology. The graphics calculator aided exploration will be able to enhance understanding of these functions and relevant problem solving.

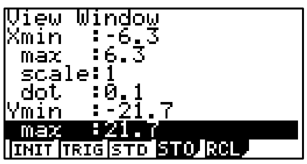
EXPLORATORY ACTIVITIES
[Note]
We shall use small letter *x*, *y* instead of capital X, Y as shown on the calculator throughout the paper.

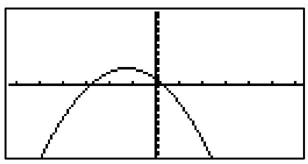
Exploration 1: Let's begin by exploring a quadratic function graphically and analyzing it with the graphics calculator. Suppose we have the function below:

$$P(x) = -2x^2 - 5x + 1.7$$

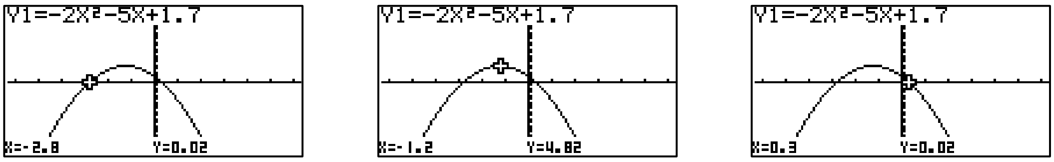
(a) Open the eActivity HIGHPOLY.g1e. Open the <u>Graph Editor</u> strip "**Exp-1A**" and graph Y1. Try graphing with the View Window setting below.



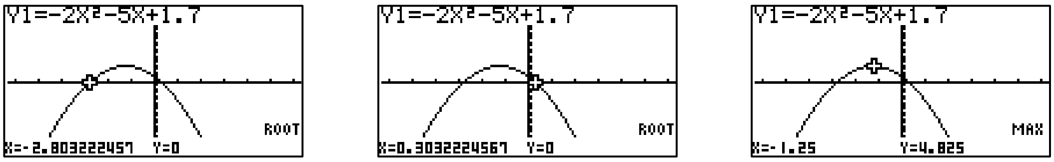




(b) Trace the graph drawn with F1 and explore in particular the x-intercepts and the vertex.

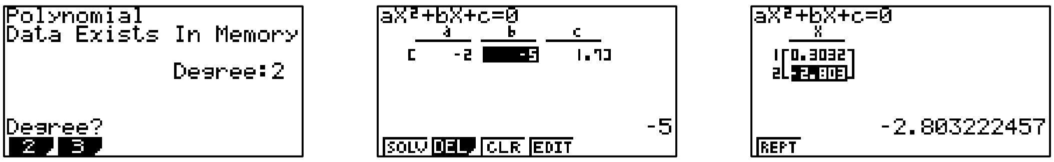


(c) Tap F5 and use the G-Solve functions to analyse this graph. Find the zeros (roots) and the visible vertex (in this case it is a maximum) of the functions.



Note that $P(x) = -2x^2 - 5x + 1.7$ has two zeros (or roots) at $x \approx$ -2.803 and 0.303 in which $P(x) = 0$. Also, this function is symmetrical where the axis of symmetry is the x-coordinate of the vertex, that is $x = -1.25$. The vertex is a maximum point whose coordinates is $(-1.25, 4.825)$.

(d) We can further complement our graphical exploration by solving the equation $P(x) = 0$ using the Polynomial function. Open the POLY strip "**Exp-1B**". Tap F1 (degree 2) to solve the quadratic equation $2x^2 - 5x + 1.7 = 0$. We should have solutions which are consistent with the zeros obtained from the graph analysis.



Discussion
With the aid of graphics calculator we can easily study the quadratic function graphically and analyse it. The graphics calculator support allows us to explore polynomial functions of higher degree.

For the function $P(x) = -2x^2 - 5x + 1.7$, its zeros are at where the x-intercepts are and could be seen in the graph. However, zeros for any polynomial functions could be complex and in the next activity we extend the exploration to find zeros of higher order polynomial functions. □

Let's look at this activity on polynomial function of degree 4.

Activity 1: Suppose we have the following degree four polynomial function

$$P(x) = 2x^4 - 7x^3 - 4x^2 + 45x - 50$$

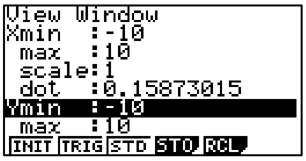
State all possible rational zeros for this polynomial function. Then, determine all rational zeros for this function. Find also the remaining zeros for $P(x)$, if there is any.

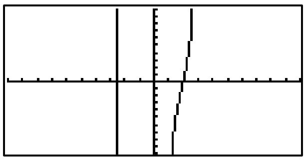
__**Solution**__:

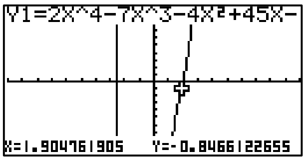
(a) From the rational zero theorem, if the rational number $\frac{a}{b}$ is a rational zero of $P(x)$, then a is a factor of 50 while b is a factor of 2. Therefore all possible rational zeros are:

$$\pm 1,\ \pm 2,\ \pm 5,\ \pm 10,\ \pm 25,\ \pm 50,\ \pm\frac{1}{2},\ \pm\frac{5}{2},\ \pm\frac{25}{2}.$$

(b) Open the Graph Editor strip "**Act1A**" and graph Y1, which is the quartic function given. Trace the graph drawn. Begin with the [STD][F3] View Window setting as displayed below.

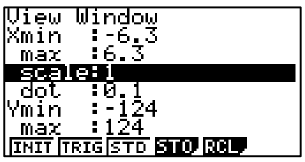


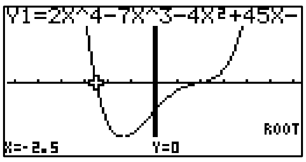


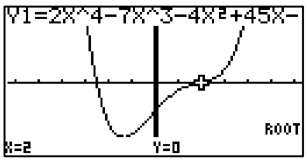


Observe that when tracing the curve, the value of y changes rapidly relative to x. However, this change in y becomes more consistent with the change of x as we trace near the positive x-intercept.

(c) Having explored the initial graph we can now set a more appropriate View Window to view the graph. Open the next <u>Graph Editor</u> strip "**Act1B**" and graph Y1. Then, analyse the graphs using [F5] (G-Solv) and determined the rational zeros for the function.







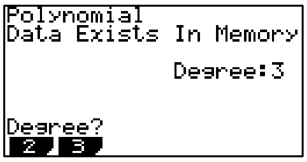
From this analysis we have found two rational zeros for $P(x)$, 2 and $-\frac{5}{2}$ (-2.5), which are consistent with the list of possible rational zeros discussed above.

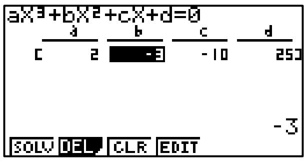
(d) To find the remaining zeros, which are not rational, we should first factorise the quartic expression with one of the rational zeros found, through synthetic division. For simplicity let's use the zero x=2.

$$\begin{array}{r|rrrrr} 2 & 2 & -7 & -4 & 45 & -50 \\ & & 4 & -6 & -20 & 50 \\ \hline & 2 & -3 & -10 & 25 & 0 \end{array}$$

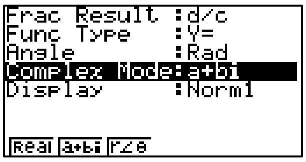
Therefore we have $P(x) = 2x^4 - 7x^3 - 4x^2 + 45x - 50 = (x-2)(2x^3 - 3x^2 - 10x + 25)$.

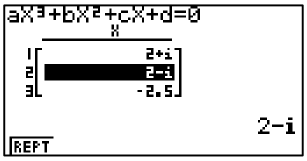
(e) Now we can find the remaining zeros by solving $2x^3 - 3x^2 - 10x + 25 = 0$. Open the <u>POLY</u> strip "**Act1C**" and tap [F2] (degree 3) to solve this cubic equation.





Tap [SHIFT] [MENU] to set up the calculator to be able to show complex zeros also. Then return to the Polynomial window to solve the equation.





We can therefore conclude that zeros of $P(x)$ are 2, -2.5, $2+i$ and $2-i$.

<u>Discussion</u>
The *n*-th root theorem says that the number of zeros a polynomial functions of degree *n* has is *n*. In the above activity we were able to obtain all four zeros for a function of degree 4. These zeros may be complex as shown in the activity, although the graph of $P(x)$ could not show the complex zeros. We can check that $P(2 \pm i)$ indeed equal to 0. □

EXERCISES

Exercise 1

State all possible rational zeros of $P(x) = 3x^3 + 2x^2 - 5x - 8$.
Graph the cubic function and by just tracing the graph drawn, state the rational zeros for the function. Then, find all zeros for the cubic function.

Exercise 2.

<u>An application</u>: The concentration of a drug called *Rudin-e* (in parts per million) in a patient's bloodstream *t* hour after the drug is administered can be represented by

$$P(t) = -t^4 + 12t^3 - 58t^2 + 132t$$

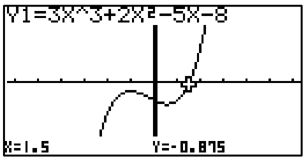
How many hours after administration will the drug are totally eliminated from the bloodstream? Also find the highest level of concentration after the drug is administered.

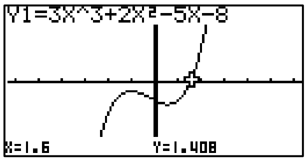
SOLUTIONS to EXERCISES

Exercise 1

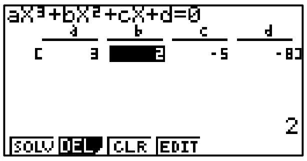
(a) If the rational number $\frac{p}{q}$ is a rational zero of $P(x)$, then *p* is a factor of 8 while *q* is a factor of 3. Hence all possible rational zeros are: ± 1, ± 2, ± 4, ± 8, $\pm\frac{1}{3}$, $\pm\frac{2}{3}$, $\pm\frac{4}{3}$, $\pm\frac{8}{3}$.

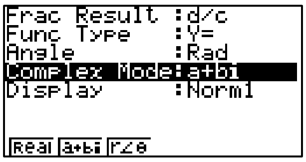
Open the Graph Editor strip "**Ex-1A**" and graph the function assigned to Y1. Tracing the graph we find that there seems to be only one rational zero (which is the x-intercept) for $P(x)$. By tracing to left and right of the x-intercept we would find that there could be a rational zero between 1.5 and 1.6. However there is no such value at the list of possible rational zeros, which implies that this function has no rational zeros.

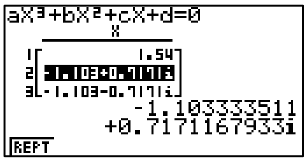




With no rational zero at hand it would not be efficient to use the approach discussed in Activity 1. Open the POLY strip "**Ex-1B**" to find the zeros for the cubic function, make sure the Complex Mode is turned on.



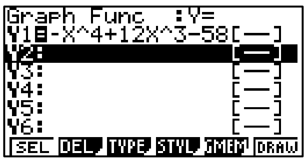


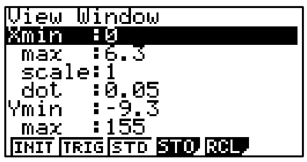


Exploring the solutions we see that this function has 1 real and 2 complex zeros, where the real zero is not rational. □

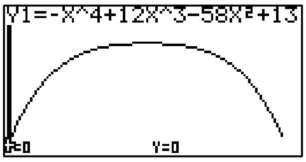
Exercise 2

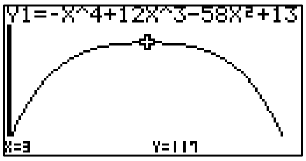
The problem can be solved by finding the real zeros for the function. Here we explore the function graphically first. Open the Graph Editor strip "**Ex-2A**" to graph Y1. Notice that we input the function using *x* to represent the variable *t*.

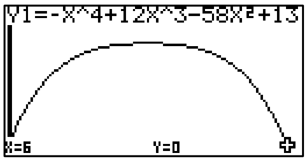




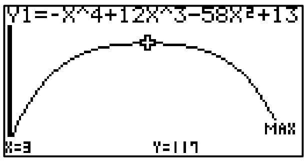
By exploring the graph through tracing, we will find that the function has real zeros of 0 and 6, with no real zero in between them. We can thus conclude that the drug will be eliminated from the bloodstream in 6 hours after its administration.







Notice also the concentration of the drug is at its highest about 3 hours after administration, which is 117 parts per million.



We can confirm the maximum concentration by analyzing the graph with [G-Solv] as shown above. □

REFERENCE

[1] Mark Dugopolski, *College Algebra and Trigonometry*, Addison Wesley, 1996. ISBN: 0-201-88952-8.

