CASIO

Cost, Revenue and Profit functions 2

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LEVEL

Middle or High school

OBJECTIVES

To study cost, revenue and profit functions with the aid of a scientific calculator.

OVERVIEW

In economics, the cost, revenue and profit functions are related by the equation: Profit= Revenue – Cost. In this paper, we study these economic quantities using algebraic and calculus concepts.

ACTIVITIES

Activity 1.

The table below contains price-demand and total cost data for the production of radial arm saws, where p is the wholesale price (in dollars) of a saw for an annual demand of x saws and C is the total cost (in dollars) of producing x saws.

x	p	C
950	240	130,000
1,200	210	150,000
1,800	160	180,000
2,050	120	190,000

a) Find the wholesale price of a saw corresponding to an annual demand of 1,400 saws.

b) How many saws were produced for a total cost of $250,000?

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c) What is the maximum profit?

Solution:

a) Our first step is to find a quadratic function $p(x)$ that will model the price demand data using x as the independent variable, and p as the dependent variable. We use a process called quadratic regression to find the function.

[Operations]

- Press Mode. Select 3: STAT [MODE] [3]
- Select 3: Quadratic Regression _ + CX^2 [3]
- Enter each of the data in the table- x values in first column, C values in the second column. After all of the data is entered, press [AC].
- Calculate the regression curve A +BX + CX^2

  Calculate the constant term A [SHIFT] [1] [7] [1] [=]

  Calculate the coefficient of X, B [SHIFT] [1] [7] [2] [=]

  Calculate the coefficient of X^2, C [SHIFT] [1] [7] [3] [=]

  Note: The values of A, B and C are to be calculated one at a time. After finding the A value, press AC and proceed to calculate B, then lastly, calculate C.

- Whenever we want to refer to the same data again, [SHIFT] [1] [2]
  The data will be kept in the calculator's memory.

We obtain $A = 288.95354$, $B = -0.0325965$ and $C = -0.00002353$. The quadratic function representing the price-demand function is given by

$$p(x) = -0.00002353x^2 - 0.0325965x + 288.95354.$$

This function models the price-demand data. This function is helpful in obtaining additional information; such as approximating price given the demand of the product, or determining the demand when price is given.

For instance, we can calculate the price corresponding to an annual demand of 1,400 saws as follows:

[Operations]

- Enter 1400 [1] [4] [0] [0]
- Calculate the dependent variable $\hat{y}$ [SHIFT] [1] [7] [6] [=]

We get an answer of 197.20. This means the wholesale price of a saw corresponding to an annual demand of 1,400 saws is $ 197.20.

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b) Now, using linear regression, we derive a linear function, that will model the cost data.

[Operations]

- Press Mode.Select 3:STAT [MODE] [3]
- Select 2: Linear Regression A+BX [2]
- Enter each of the data in the table- x values in first column, p values in the second column. After all of the data is entered, press [AC].
- Calculate the regression line A +BX
  Calculate the constant term A [SHIFT] [1] [7] [1] [=]
  Calculate the coefficient of X, B [SHIFT] [1] [7] [2] [=]

We obtain $A = 82245.2229$ and $B = 82245.2229$. The linear function that models the cost function is $C(x) = 53.5031847x + 82245.2229$.

To determine the number of saws that are produced for a total cost of \$250,000, we use the following calculator commands:

[Operations]

- Enter 250000 [2] [5] [0] [0] [0] [0]
- Calculate the independent variable $\hat{x}$ [SHIFT] [1] [7] [6] [=]

The answer we get is 3135.41. This means that the approximate number of saws that are produced for a total cost of \$ 250,000 is approximately 3135.

c). From a), the price demand function is $p(x) = -0.00002353x^2 - 0.0325965x + 288.95354$ and from b), the cost function is $C(x) = 53.5031847x + 82245.2229$. The revenue function is given by $R(x) = p(x)x$ where $p(x)$ is the price demand function. The profit function is now calculated as follows:

$$P(x) = R(x) - C(x) = p(x)x - C(x)$$

$$= (-0.00002352x^2 - 0.0325964x + 288.95354)\,x - (53.5031847x + 82245.2229)$$

$$= -0.00002352x^3 - 0.0325964x^2 + 288.95354x - 53.5031847x - 82245.2229$$

$$= -0.00002352x^3 - 0.0325964x^2 + 235.4503553x - 82245.2229$$

To calculate maximum profit, we solve $P'(x) = 0$.

Now, $P'(x) = (3)(-0.00002352)x^2 - (2)(0.0325964)x + 235.450355$ or equivalently,

$P'(x) = -0.00007056x^2 - 0.0651928x + 235.450355$.

To find x such that $P'(x) = 0$ we use the following calculator commands:

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[Operations]

- Press Mode. Select 5: EQN — MODE 5
- Select 3, ax^2+bx+c — 3
- Enter a, $(3)(-0.00002352)$ — 3 × (−) 0 • 0 0 0 0
- Continue — 2 3 5 2 =
- Enter b, $-(2)(0.0325964)$ — (−) 2 × 0 • 0 3 2 5 9 6 4 =
- Enter c, 235.450355 — 2 3 5 • 4 5 0 3 5 5 =
- Enter equals sign to get first root — =
- Enter equals sign again to get next root — =

The solutions to $P'(x) = -0.00007056x^2 - 0.0651928x + 235.450355 = 0$ are $x_1 = 1422.26$ and $x_2 = 2346.19$.

We now verify $P(1422.26)$ and $P(2346.19)$ to determine the maximum profit. We illustrate how to obtain $P(1422.26)$ using the calculator. We do a similar process for $P(2346.19)$.

[Operations]

- Press Mode. Select 1: COMP — MODE 1
- Enter $-0.00002352(1422.26)^3$ — (−) • 0 0 0 0 2 3 5 2
- Continue — (1 4 2 2 • 2 6) $x^{\blacksquare}$ 3
- Enter $-0.0325964(1422.26)^2$ — − • 0 3 2 5 9 6 4
- Continue — (1 4 2 2 • 2 6) x^2
- Enter + $235.4503553(1422.26)$ — + 2 3 5 • 4 5 0 3 5 5 3
- Continue — (1 4 2 2 • 2 6)
- Enter -82245.2229 — − 8 2 2 4 5 • 2 2 2 9

We obtain $P(1422.26) = 119{,}023.043$ and $P(2346.19) = -13021.73$. Thus, the maximum profit is given to be approximately \$119,023 and this occurs when the annual demand of saws is 1422.26 units.

Activity 2.

The financial department of a company that produces automatic cameras arrived at the following revenue and cost functions:

$$R(x) = 94.8x - 5x^2 \text{ and } C(x) = 156 + 19.7x \text{ , } 1 \le x \le 15$$

a) Find the output that will produce maximum revenue.

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b) Find the break even points.

c) For what outputs will a loss occur? A profit?

d) Find the output that will give a maximum profit.

Solution:

a) Using calculus we obtain maximum revenue by solving $R'(x) = 94.8 - 10x = 0$. We obtain $x = \frac{94.8}{10} = 9.48$. Thus, producing about 10 cameras will give maximum revenue.

Using an algebraic approach, maximum revenue will occur at an x value midway between the roots of $R(x) = 94.8x - 5x^2$.

We find the roots of $R(x) = 94.8x - 5x^2$ using the calculator as follows:

[Operations]

- Press Mode. Select 5: EQN 
- Select 3, ax^2+bx+c 
- Enter a, -5 
- Enter b, 94.8 
- Enter equals sign to get first root 
- Enter equals sign again to get next root 

The roots of $R(x) = 94.8x - 5x^2$ are $x_1 = 18.96$ and $x_2 = 0$. Thus, maximum revenue will occur at $x = \frac{x_1 + x_2}{2} = \frac{18.96}{2} = 9.48$.

b) The break-even points are the production levels at which $R(x) = C(x)$ or equivalently, the values of x where there is no profit or $P(x) = 0$. Solving $94.8x - 5x^2 = 156 + 19.7x$ or $75.1x - 5x^2 - 156 = 0$, we find the roots of $75.1x - 5x^2 - 156$ to be: $x_1 = 2.49$ and $x_2 = 12.53$. We use the same calculator keystrokes given in a) to obtain the answers.

c) A loss occurs if $P(x) < 0$ and a profit occurs if $P(x) > 0$. Now, $P(x) > 0$ in the interval $[2.49, 12.53]$ where the break even points occur at $x_1 = 2.49$ and $x_2 = 12.53$. Moreover, $P(x) < 0$ in the intervals $[1, 2.49]$ and $[12.53, 15]$.

d) The profit function is given by $P(x) = R(x) - C(x) = 75.1x - 5x^2 - 156$. Solving $P'(x) = 75.1 - 10x = 0$, we obtain $x = 7.51$. Thus, an output of approximately 8 cameras will yield maximum profit.

Observe that the break even points are the x-intercepts of the profit function. We can verify that the maximum profit occurs at the output level midway between the break-even points. Note that $\frac{x_1 + x_2}{2} = \frac{2.49 + 12.53}{2} = 7.51$.

ASSESSMENT TASKS

Exercise .

The research department in a company that manufactures clock radios established the following cost and revenue functions:

$$C(x) = 160 + 10x\ ,\ R(x) = 50x - 1.25x^2$$

a) Determine the maximum revenue and the output that produces the maximum revenue.

b) Find the break even points.

c) Does the maximum profit appear to occur at the same output level as the maximum revenue?

Solution:

a) Using calculus, $R'(x) = 50 - 2.50x = 0$ implies $x = \frac{50}{2.5} = 20$. Thus, producing about 20 radios will give maximum revenue.

b)The break-even points are obtained from $R(x) = C(x)$. Now, $50x - 1.25x^2 = 160 + 10x$ implies $40x - 1.25x^2 - 160 = 0$. The solutions to this equation are $x_1 = 27.31$ and $x_2 = 4.69$.

c) Note that $\frac{x_1 + x_2}{2} = \frac{27.31 + 4.69}{2} = 16$. Thus, maximum profit will occur at production level of 16 radios. Maximum profit does not occur at the same output where maximum revenue occurs.

REFERENCE

Calculus for Business, Economics, Life Sciences and Social Sciences, Ninth Edition, Barnett, et al, 2003, Pearson Education Asia, Singapore.

