



Applications to Integration

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LEVEL

High school or university students who have studied the integration.

OBJECTIVES

To use calculator to explore real-life problems in integration theory.

OVERVIEW

- (i) We will use calculator to explore the annuity problem.
- (ii) We will use calculator to explore the integration of even and odd functions.

EXPLORATORY ACTIVITIES

[Note]

(a) We shall use small letter x instead of capital X as shown on the calculator throughout the paper.

(b) Unless otherwise specified, we choose MATH mode in the SET UP menu, using

SHIFT **MODE** **1** (MthIO)

A sequence of equal payments made at regular time intervals over a period of time is called an **annuity**. For example, annuity could be payroll savings plans, monthly home mortgage payments and IRA (Individual retirement accounts). The amount of annuity is the sum of the payments plus the interest earned and if $c(t)$ represents a continuous income function in dollars per year (where t is the time in years), r represents the interest rate compounded continuously, and T represents the term of the annuity in years, then **the amount of an annuity** is given by

$$e^{rT} \int_0^T c(t)e^{-rt} dt.$$

Activity 1: If you deposit \$2,000 each year for 10 years into an IRA paying 6% interest. How much will you have in your IRA after 10 years?









Solution:

The income function $c(t)=2000$. The amount of annuity after 10 years will be

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$$e^{(0.06)(10)} \int_0^{10} 2000e^{-0.06t} dt = 27403.96.$$

[Operations]

- Enter the computation Mode. 
- Enter $e^{(0.06)(10)}$ 
- Enter $x \int_0^{10} 2000e^{-0.06x} dx$. 
- Continue 
- Continue 
- Lower limit 
- Upper limit 
- Press equal sign 

We get the answer \$27,403.96..

Next, we explore integration over an even function, (when the graph of such function is symmetric with respect to y-axis or when $f(-x)=f(x)$) and an odd function (when the graph of such function is symmetric to the origin $(0,0)$ or when $f(-x)=-f(x)$).

If a is any positive real number, we have the followings:

(1) If f is an even function, then $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$.

(2) If f is an odd function, then $\int_{-a}^a f(x)dx = 0$.

Remark:

(1) It will be easier to find $\int_{-a}^a f(x)dx$ if a function can be decomposed into the combinations (sum or difference) of two even and odd functions.

(2) This will be quite useful once students learn about the Maclaurin series for a function.

For example, if we try to approximate $\int_{-0.1}^{0.1} \ln(x+1)dx$ and if we can show that

$$\ln(x+1) \approx \frac{-x^4}{4} + \frac{x^3}{3} - \frac{x^2}{2} + x.$$

Then $\int_{-0.1}^{0.1} \ln(x+1)dx$ can be reduced to $2\int_{0.1}^{0.1} \left(\frac{-x^4}{4} - \frac{x^2}{2}\right)dx$ since $\int_{-0.1}^{0.1} \left(\frac{x^3}{3} + x\right)dx$ is 0, we will demonstrate this in the next Activity.

Activity 2: (a) Find $\int_{-0.1}^{0.1} \ln(x+1)dx$ directly. (b) Find $2\int_{-0.1}^{0.1} \left(\frac{-x^4}{4} - \frac{x^2}{2}\right)dx$ (c) Find

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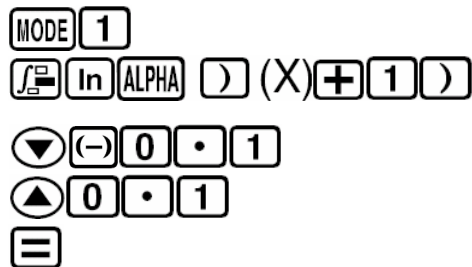
$$\int_{-0.1}^{0.1} \left(\frac{x^3}{3} + x \right) dx.$$

Solution:

(a)

[Operations]

- Enter the computation Mode.
- Enter $\int_{-0.1}^{0.1} \ln(x+1) dx$
- Enter the lower limit
- Enter the upper limit
- Press equal sign

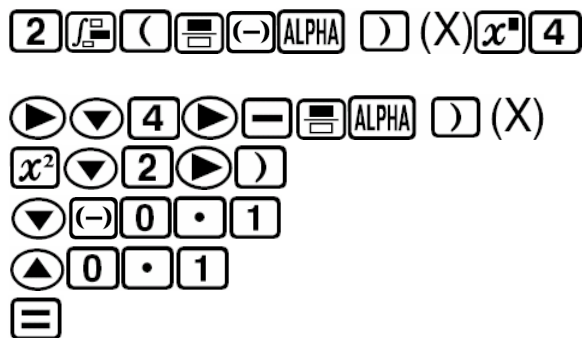


We get $\int_{-0.1}^{0.1} \ln(x+1) dx = -0.00033$.

(b)

[Operations]

- Enter $2 \int_{-0.1}^{0.1} \left(\frac{-x^4}{4} - \frac{x^2}{2} \right) dx$
- Continue
- Continue
- Enter the lower limit
- Enter the upper limit
- Press equal sign

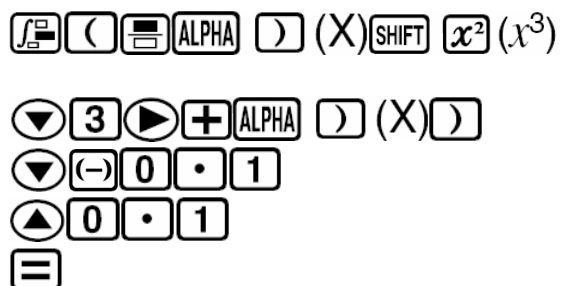


We get $2 \int_{-0.1}^{0.1} \left(\frac{-x^4}{4} - \frac{x^2}{2} \right) dx = -0.00067$.

(c)

[Operations]

- Enter $\int_{-0.1}^{0.1} \left(\frac{x^3}{3} + x \right) dx$
- Continue
- Enter the lower limit
- Enter the upper limit
- Press equal sign



We get $\int_{-0.1}^{0.1} \left(\frac{x^3}{3} + x \right) dx = 0$.

Activity 3: If $f(x) = \sin(2x) - 2x$. (a) Determine if f is even or odd. (b) Find $\int_{-100}^{100} f(x)$.

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Solution:

For (a), we check (by hand) $f(-x)$ first. In this case, since

$$f(-x) = \sin(-2x) + 2x = -\sin(2x) + 2x = -(\sin(2x) - 2x) = -f(x),$$

we see this function is odd. We can also explore the table for $y=f(x)$, which we show below.

[Operations]

- Enter the Table Mode.
- Enter $\sin(2x) - 2x$.
- Continue
- Give a starting value, say -10.
- Give a ending value, say 10.
- We the step size, say 1

MODE 7
 sin 2 ALPHA) (X)) - 2
 ALPHA) (X) =
 (-) 1 0 =
 1 0 =
 1 =

X	F(X)
-10	19.65798
-9	17.69098
-8	15.72436
-7	13.75808
7	-13.75808
8	-15.72436
9	-17.69098
10	-19.65798

The above partial table confirms that this function is odd.

(b) We expect $\int_{-100}^{100} f(x) dx$ to be 0.

[Operations]

- Enter the computation Mode.
- Enter $\int_{-100}^{100} (\sin 2x - 2x) dx$
- Continue
- Enter the lower limit
- Enter the upper limit
- Press equal sign

MODE 1
 \int_{\square} sin 2 ALPHA) (X)) - 2
 ALPHA) (X)
 ▼ (-) 1 0 0
 ▲ 1 0 0
 =

EXERCISES

Exercise 1. Follow the Activity 1. Find the amount of annuity with income function $c(t)=1500$, interest rate $r=2\%$, and term $T=10$ years.

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Exercise 2. Follow the Activity 1. Find the amount of annuity with income function $c(t)=250$, interest rate $r=8\%$, and term $T=6$ years.

Exercise 3. If $f(x) = \cos(x^2 - 1) + 2x^4$. (a) Determine if f is even or odd. (b) Find

$$\int_{-1}^1 f(x).dx$$

SOLUTIONS

Exercise 1

\$16,605.21

Exercise 2.

\$1,925.23.

Exercise 3.

The function is even and $\int_{-1}^1 f(x).dx = 2 \int_0^1 f(x).dx = 2.79984$

REFERENCE

Brief Calculus-An Applied Approach, by Larson, Edwards, Falvo, Houghton Mifflin Company, 2003, six ed., ISBN 0-618-21870-X.