Simultaneous Linear Equations 1

Barry Kissane
Murdoch University, WA
Australia

LEVEL
High school after students have learned about linear equations

OBJECTIVES
(i) To understand the substitution method of solving a pair of simultaneous linear equations

(ii) To use the automatic solver to solve a pair of simultaneous linear equations.

OVERVIEW
Pairs of simultaneous linear equations occur frequently and students need to understand various solution methods. In the first part of this activity, the substitution method is explained and the table function of the calculator used to understand how it works. In the second part of the activity, the automatic solver of the calculator is used, for efficient solutions to a system. The exercises encourage students to understand some of the possible kinds of pairs of equations and also to develop expertise with calculator use for tables and equations.

EXPLORATORY ACTIVITIES
[Note]
(a) We shall use small letter x instead of capital X as shown on the calculator throughout the paper.
(b) Unless otherwise specified, we choose MATH mode in the SET UP menu, using
SHIFT MODE 1 (MthIO)

Activity 1: The substitution method

Consider the following pair of linear equations:

$2x + y = 7$
$4x + 3y = 5$

To *solve the equations simultaneously* means that a pair of values (x,y) that satisfies *each* of the two equations must be found. It is not always possible to do this.
One way of solving this linear system is called the *substitution method*. We will use it in this activity.

The solution must satisfy both relationships, $2x + y = 7$ and also $4x + 3y = 5$.

Rearrange the first equation to obtain $y = 7 – 2x$. Then substitute this relationship for y into the second equation:

$$4x + 3y = 5$$

Becomes:

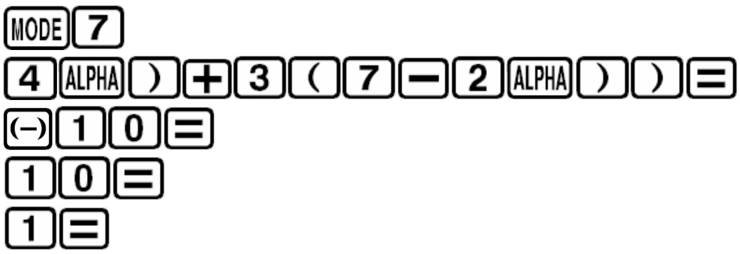
$$4x + 3(7 – 2x) = 5$$

Although this may look more complicated, we can find a solution now by solving this new equation for x, since it has only one variable.

To find the solution, we will use the Table function of the calculator to evaluate the left hand side, 4x + 3(7 – 2x), for several values of x, to see whether there is a value for which the expression equals 5.

[Operations]

- Press Mode. Select 7: TABLE
- Enter $f(x) = 4x + 3(7 – 2x)$.
- Start with the value -10.
- End with the value 10.
- Use 'Step' 1.



The calculator displays a table of x and *f(x).* Use the cursor key  to display the values of $4x + 3(7 – 2x)$ for various values of x. Use the cursor key  to scroll down to find the value of x for which $4x + 3(7 – 2x) = 5$. This value is $x = 8$, which shows that the two equations are both satisfied for $x = 8$.

To find the value for y, substitute $x = 8$ into one of the original equations. The easiest one to choose is $y = 7 – 2x$, giving the result $y = 7 – 2(8) = -9$. So $x = 8$ and $y = -9$ is a solution to the pair of equations. Check for yourself that, for these values, $4x + 3y = 5$.
Notice that other substitutions might have been chosen in this case. For example, the second equation leads to the substitution:

$Y = (5 – 4x)/3$

Some of these possibilities are explored in the exercises.

Activity 2: Using the solver

Consider the following pair of linear equations:

$2y = 8 - 5x$
$2x + 3y - 5 = 0$

An alternative to using the substitution method to solve these simultaneously is to use the automatic equation solver in the calculator. To use the solver, you must first rearrange the equations in the necessary form. For a pair of simultaneous equations, the form is shown by the calculator as:

$a_n X + b_n Y = c_n$

The *x* term comes first, the *y* term comes second and the constant is after the equals sign. In this case, the two equations need to be rearranged as:

$5x + 2y = 8$
$2x + 3y = 5$

When the equations are in this form, the values of the *coefficients*, are seen to be a = 5, b = 2 and c = 8 for the first equation, with a = 2, b = 3 and c = 5 for the second equation.

The following steps show the solution of the system of equations.

[Operations]

- Press Mode. Select 5: EQN — MODE 5
- Select 2-variables — 1
- Enter coefficients: — 5 = 2 = 8 =
- Enter coefficients — 2 = 3 = 5 =
- Press = to get *x* solution — =
- Press = to get *y* solution — =

The solution is $x = 14/11$ and $y = 9/11$. Check these by substituting the values into one of the equations.

After using the solver, press the = key to change the coefficients or the MODE key to perform a different calculation.

EXERCISES

The purpose of the exercises is to give you an opportunity to do some independent work, to develop fluency with calculator use and to learn more about solutions of pairs of simultaneous equations.

Simultaneous Linear Equations 1

Exercise 1.
Consider again the following pair of linear equations:

$2x + y = 7$
$4x + 3y = 5$

(a) Use the substitution method to solve the equations, getting the same solution as above. This time, however, use the substitution $y = (5 - 4x)/3$ from the second equation. Notice that the procedure is different, although the answer is the same.

(b) Which choice for substitution is better: $y = (5 - 4x)/3$ or $y = 7 – 2x$? Give a reason for your preference.

Exercise 2
Consider again the following pair of linear equations:

$2x + y = 7$
$4x + 3y = 5$

(a) It is possible to substitute for x instead of y. Use the substitution $x = (7 – y)/2$ to solve the equations. You should get the same result as previously.

(b) What other substitution for x might have been used in this case?

Exercise 3
Consider again the following pair of linear equations:

$2y = 8 - 5x$
$2x + 3y – 5 = 0$

Use the substitution method to solve the equations, getting the same solution as above.

Exercise 4
Consider the following pair of linear equations:

$2x + y = 8$
$4x + 3y = 5$

(a) Use the substitution method to solve the equations. Notice that it is necessary to choose a step that is not a whole number in order to do this.

(b) Solve the equations using the automatic solver. Check that your solutions are the same as those obtained by using substitution.

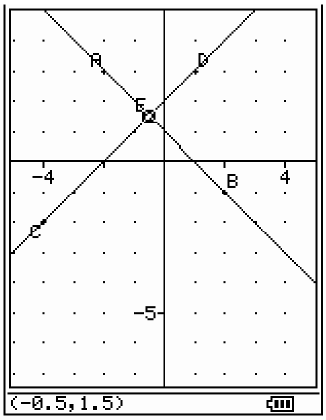
Exercise 5
Consider the following system of linear equations:

$5x + y = 8$
$4x + 3y = 5$

(a) Solve the system using substitution. Check your answers by using the automatic solver.

(b) Explain why the solution using the solver is much easier than that using substitution.

Exercise 6
Another way to solve pairs of simultaneous equations is to use graphs. Consider the screen below from a *ClassPad 300*:



The screen shows line AB with equation $x + y = 1$ and line CD with equation $y = x + 2$. Solve these two equations simultaneously and explain how your answer is related to the graphs showing on the screen.

Exercise 7
Some systems of equations do not have solutions. Use the automatic solver to see which of the following systems do not seem to have solutions:

(a)	$x + 2y = 8$ $x - 3y = 5$	(b)	$x + 2y = 8$ $2x + 4y = 16$	(c)	$4x = 5 - 7y$ $8y = 3 - x$
(d)	$5x + y - 8 = 0$ $3x - 5y - 5 = 0$	(e)	$y - 5x = 9$ $x + y = 6$	(f)	$2x + 7y = 5$ $2x + 7y = 11$

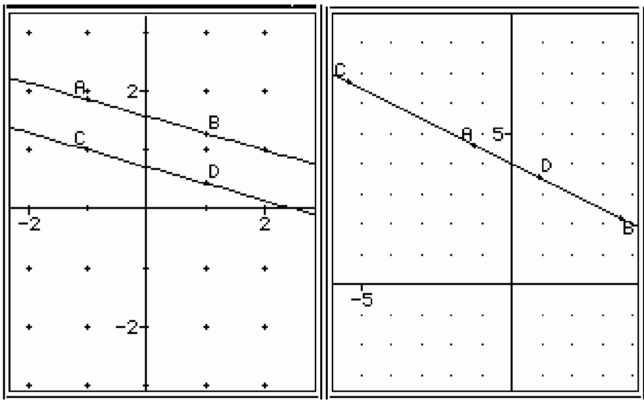
Exercise 8
Consider carefully the pairs of equations in Exercise 7(b) and 7(f).

(a) Explain why solutions to the first equation in 7(b) are also solutions to the second equation. Hence give several solutions to the system of equations in Exercise 7(b).

(b) Explain why solutions to the first equation in Exercise 7(f) cannot be solutions to the second equation. Hence explain why there are no solutions to the system of equations in Exercise 7(f).

(c) Use substitution to solve the systems in Exercise 7(b) and Exercise 7(f). Explain what you observe.

(d) Each screen below show a pair of lines AB and CD corresponding to the two equations in Exercise 7(b) and Exercise 7(f).



Identify which screen shows Exercise 7(b) and which screen shows Exercise 7(f). Then use the graphs to justify your explanations above regarding the two systems of equations.

SOLUTIONS

Exercise 1
(a) Solve 2*x* + (5 – 4*x*)/3 = 7
(b) *y* = 7 – 2*x* is easier to use, although each gives the same result.

Exercise 2
(b) *x* = (5 – 3*y*)/4

Exercise 3
Substitute *y* = (8 – 5*x*)/2 and then solve 2*x* + 3(8 – 5*x*)/2 = 0

Exercise 4
(a) Substitute *y* = 8 – 2*x* and then solve 4*x* + 3(8 – 2*x*) = 5. Need to use Step = ½ to get *x* = 19/2 and thus *y* = -11.

Exercise 5
(a) Substitute *y* = 8 – 5*x* and then solve 4*x* + 3(8 – 5*x*) = 5 to get *x* =19/11 and *y* = -7/11.
(b) Substitution requires a step of 1/11 in the table, which is very hard to see.

Exercise 6
Solution is *x* = -1/2, *y* = 3/2. These are the coordinates of the point of intersection of the two lines.

Exercise 7
Only (b) and (f) do not have solutions.

Exercise 8

(a) If $x + 2y = 8$, then $2(x + 2y) = 2x + 4y = 16$, so that any pair of numbers satisfying one equation also satisfies the other equation. Some possible solutions are $x = 2$, $y = 3$; $x = 4$, $y = 2$; $x = 8$, $y = 0$; etc.
(b) If $2x + 7y = 5$, then $2x + 7y$ cannot also equal 11, so there are no solutions.
(c) For 7(b), substitute $x = 8 - 2y$ to get $2(8 - 2y) + 4y = 16$, for all values of the variable. For 7(f), substitute $x = (5 - 7y)/2$ to get $2(5 - 7y)/2 + 7y = 5$, regardless of the value of the variable, so can never equal 11. Hence there is no solution.
(d) Exercise 7(b) is shown on the right. The two equations have the same line, so that any solution to one equation will also be a solution to the other. Exercise 7(f) is on the left. There are no points (x,y) satisfying both equations.

